Quasi-Monte Carlo Methods

Quasi-Monte Carlo Simulations

Quasi-Monte Carlo simulations are variations of Monte Carlo simulations.

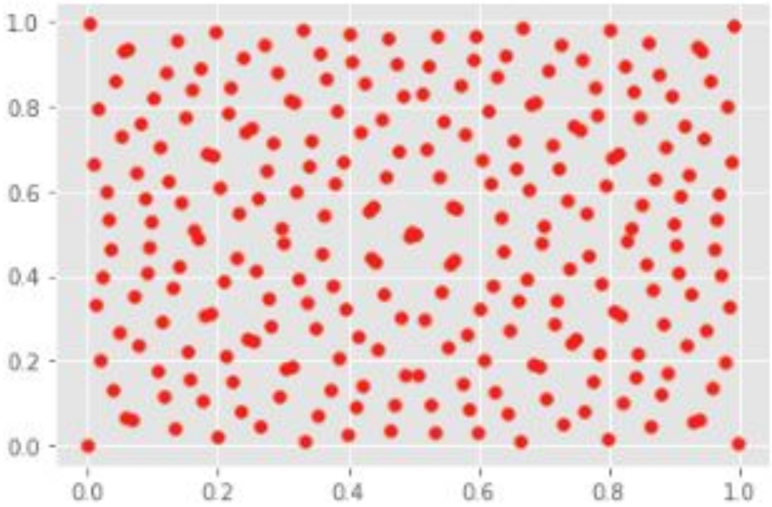
It uses **low-discrepancy deterministic sequences** with **better uniformity** properties than purely random sequences providing **more accurate and efficient estimates** compared to traditional Monte Carlo simulations.

Example: **Halton** or **Sobol** sequences.

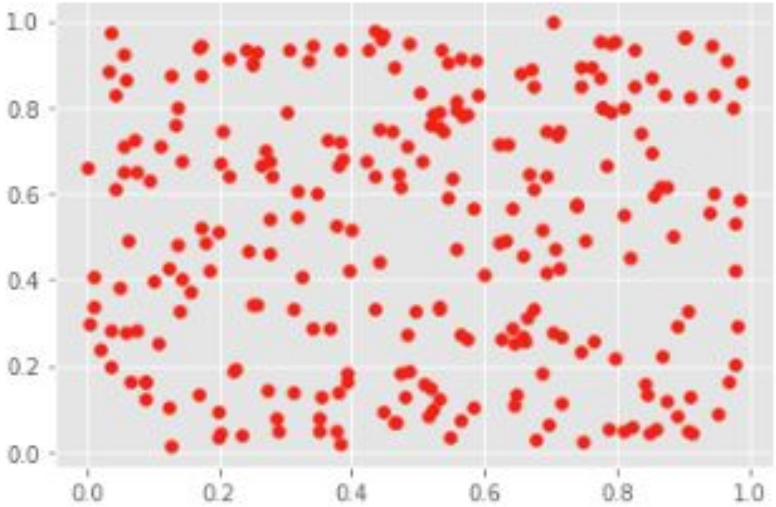
Monte Carlo vs Quasi-Monte Carlo Simulations

We generate below 256 points uniformly distributed in $[0,1]^2$ with Monte Carlo and Quasi-Monte Carlo (Sobol sequence) methods.

Quasi-Monte Carlo



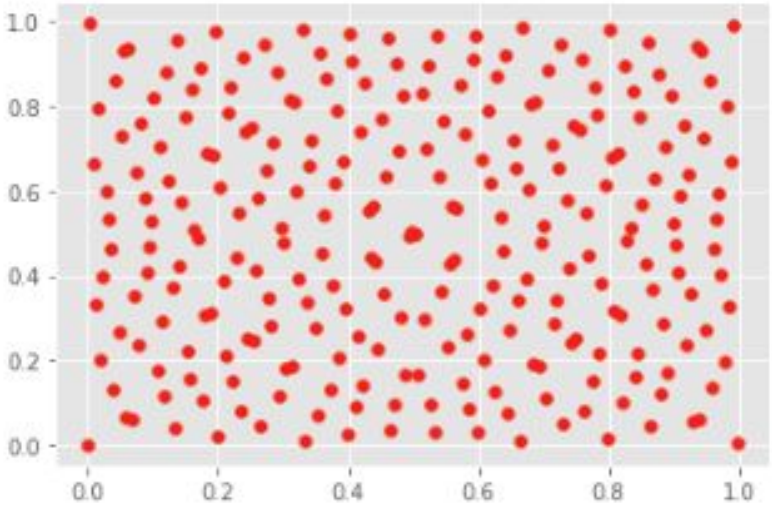
Monte Carlo



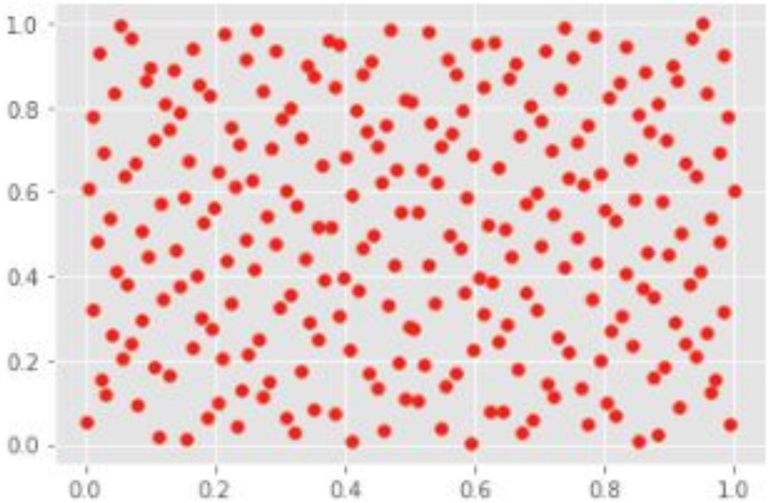
Monte Carlo vs Quasi-Monte Carlo Simulations

We generate below 256 points uniformly distributed in $[0,1]^2$ with Monte Carlo and Quasi-Monte Carlo methods.

Quasi-Monte Carlo



Quasi-Monte Carlo with Scrambling



Monte Carlo vs Quasi-Monte Carlo Simulations

Quasi-Monte Carlo allows more accurate and efficient estimates with a rate of convergence close to $O(\log(N)^k N^{-1})$ for a problem of dimension k compared to $O(N^{-0.5})$ with Monte Carlo simulations.

We would get roughly the same accuracy of the estimate with 10^4 quasi-Monte Carlo simulations than with 10^7 Monte Carlo simulations for a problem of dimension 1!!

Monte Carlo vs Quasi-Monte Carlo Simulations: Pi Estimation

```
import numpy as np
from scipy.stats import qmc
import matplotlib.pyplot as plt
plt.style.use('ggplot')
```

```
def MC_Pi(n):
    x1=np.random.uniform(-1,1,n)
    x2=np.random.uniform(-1,1,n)

    y = x1[np.sqrt(x1**2+x2**2)<1]
    pi_est = len(y) / n * 4

    return pi_est
```

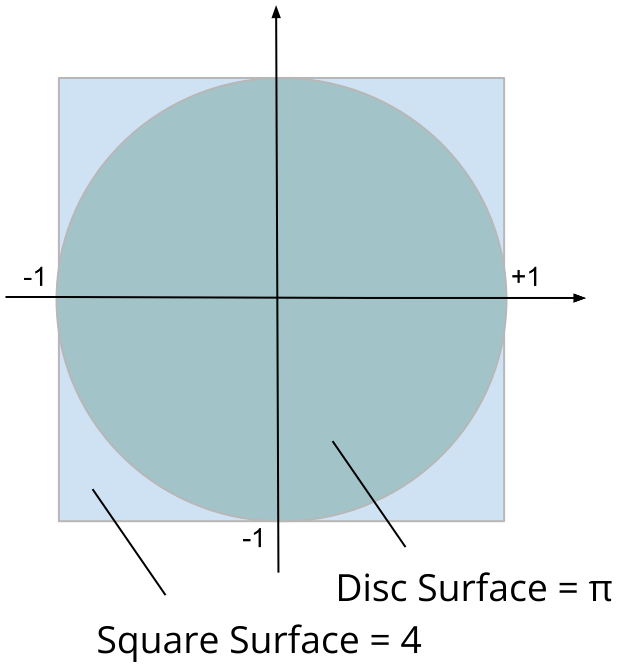
```
def QMC_Pi(n):

    sobol = qmc.Sobol(d = 2, scramble = True) #Sobol sequence
    x = sobol.random(2**n) # Generate Quasi-Monte Carlo random numbers

    x1 = x[:,0]
    x2 = x[:,1]

    y = x1[np.sqrt(x1**2+x2**2)<1]
    pi_est = len(y) / 2**n * 4

    return pi_est
```



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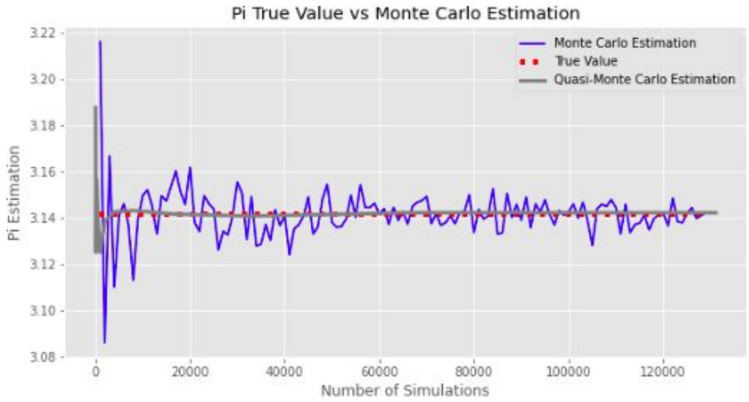
    return pi_est
```

```
Pi_MC = MC_Pi(2**20)
Pi_QMC = QMC_Pi(20)
print("Pi MC Estimation: " + str(np.round(Pi_MC, 6)))
print("Pi QMC Estimation: " + str(np.round(Pi_QMC, 6)))
print("Pi True Value: " + str(np.round(np.pi, 6)))
```

```
Pi MC Estimation: 3.140198
Pi QMC Estimation: 3.141491
Pi True Value: 3.141593
```



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website: **www.quant-next.com**

email: **contact@quant-next.com**

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