How the Heston Parameters Control the Implied Volatility Surface

The Heston Model
Risk-Neutral Probability

Under the risk-neutral probability Q it can be expressed as[1]:

$$dS_t = r \cdot S_t \cdot dt + \sqrt{\nu_t} \cdot S_t \cdot dW_t^{1,Q}$$

$$d\nu_t = \kappa^Q \cdot (\theta^Q - \nu_t) \cdot dt + \xi \cdot \sqrt{\nu_t} \cdot dW_t^{2,Q}$$

$$dW_t^{1,Q} \cdot dW_t^{2,Q} = \rho \cdot dt$$

The Heston Model has five unknown parameters:

$$\left(\nu_0, \kappa^Q, \theta^Q, \xi, \rho\right)$$

S_t : Underlying asset price at time t

r : Risk-free interest rate

ν_t : Instantaneous variance at time t

κ^Q : Speed of mean-reversion (Q)

θ^Q : Long-term mean of the variance (Q)

ξ : Volatility of the variance (vol of vol)

$W_t^{1,Q}, W_t^{2,Q}$: Wiener processes under Q

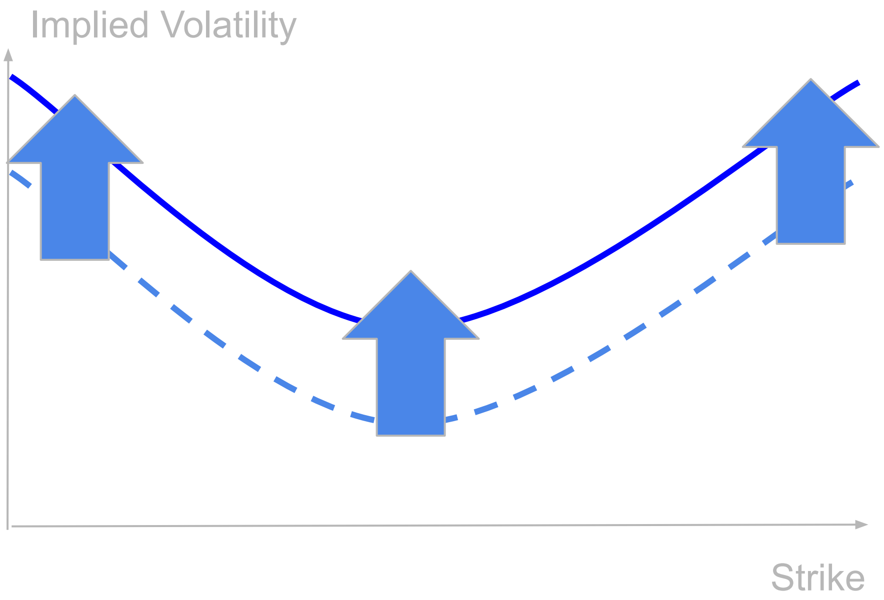
ρ : Correlation of the two Wiener processes

[1]See Heston, Steven L. (1993), “A closed-form solution for options with stochastic volatility with applications to bond and currency options”.

Volatility Smile and Skew in The Heston Model

The **initial variance v_0** and the **long-term mean of the variance θ^Q** control the **level** of the implied volatility curve.

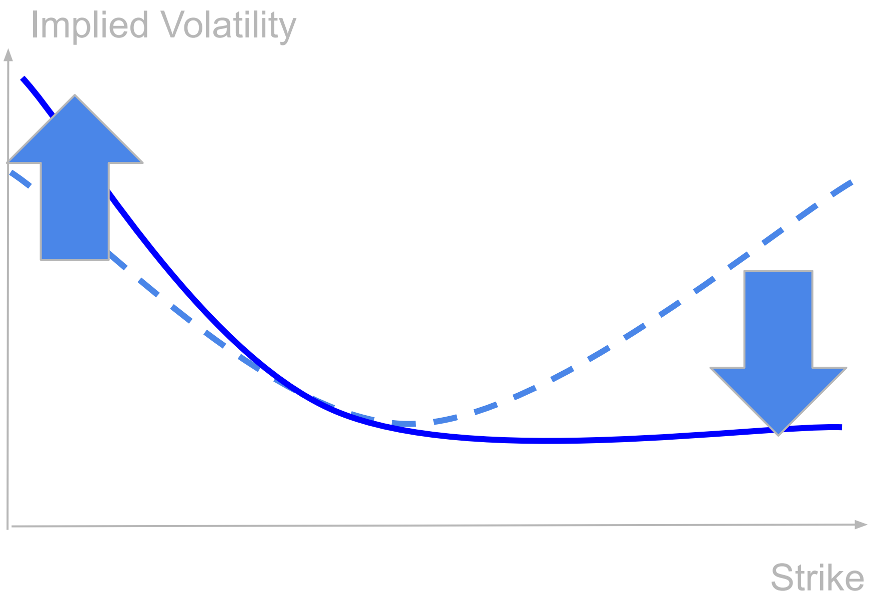
They control the **second moment**, the **variance**, of the underlying asset return distribution implied by option prices.



Volatility Smile and Skew in The Heston Model

The **spot / vol correlation ρ** controls the **slope**, the **skew** of the implied volatility curve.

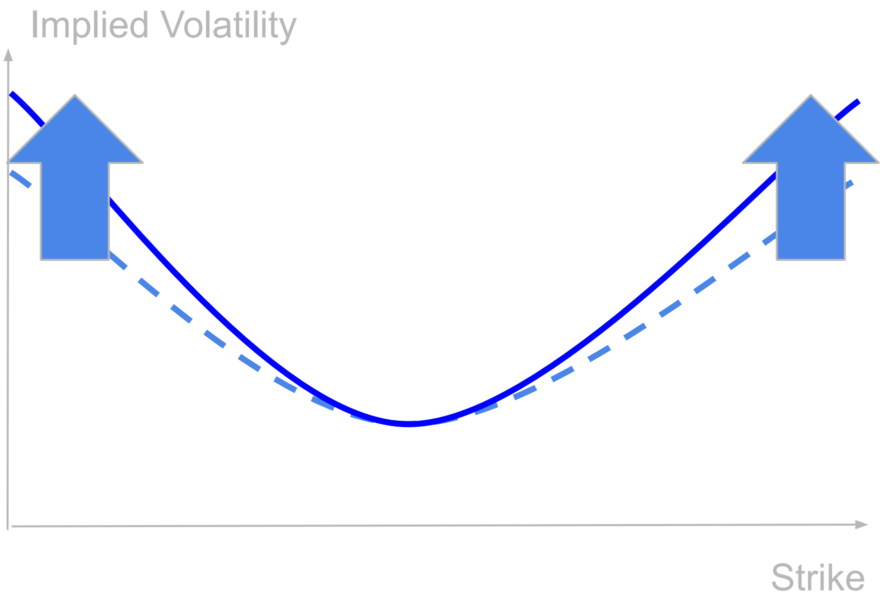
It controls the **third moment**, the **skewness**, of the underlying asset return distribution implied by option prices.



Volatility Smile and Skew in The Heston Model

The **vol of vol ξ** controls the **smile** of the implied volatility curve.

It controls the **fourth moment**, the **kurtosis**, of the return distribution implied by option prices.



Variance Swap Term Structure In the Heston Model

In the Heston model, the cumulative variance is the integral of the instantaneous variance, which follows a Cox-Ingersoll-Ross (CIR) model.

$$V_t = \int_0^t \nu_s ds$$

It can be shown[1] that, the expected annualized variance for a time horizon T is given by

$$\frac{1}{T} \cdot E(V_T) = \theta^Q + \frac{1-e^{-\kappa^Q \cdot T}}{\kappa^Q \cdot T} \cdot (\nu_0 - \theta^Q)$$

[1]See Guillaume F., Shoutens W. (2013), “Heston Model: the Variance Swap Calibration”.

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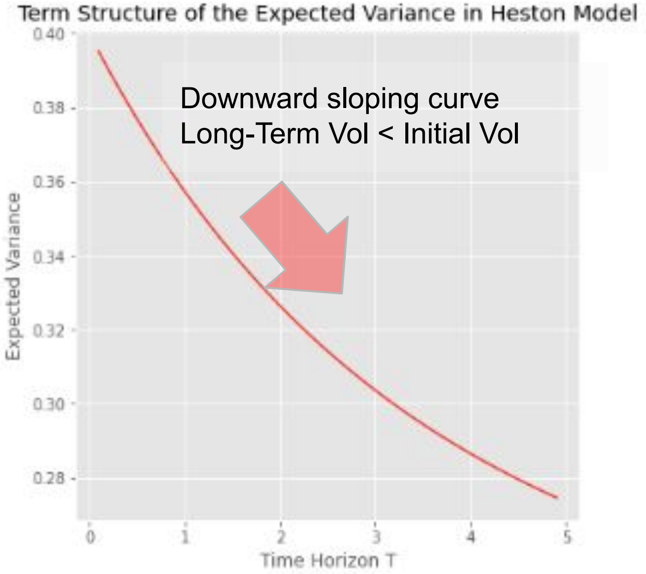

$$\nu_0 = 0.4, \theta^Q = 0.2, \kappa^Q = 0.5$$

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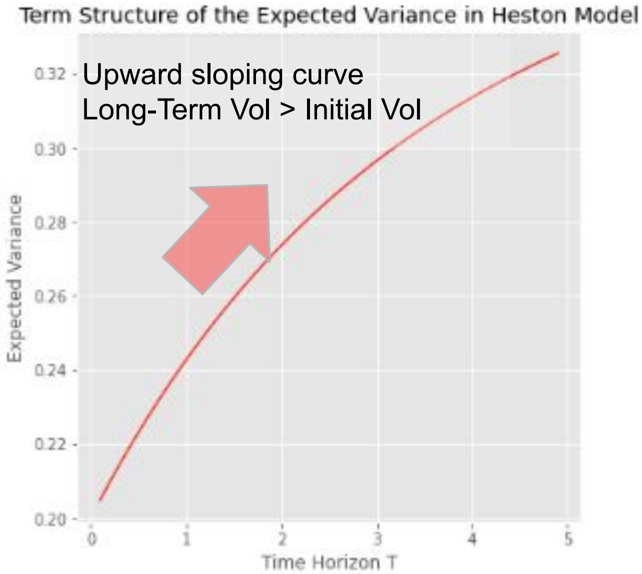

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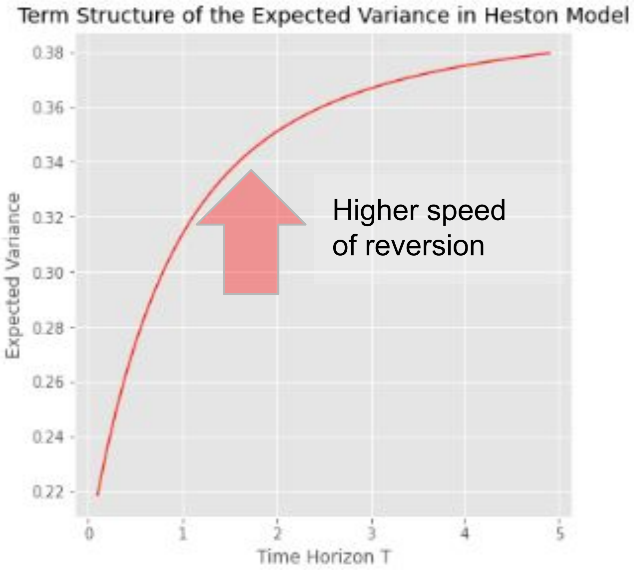

$$\nu_0 = 0.2, \theta^Q = 0.4, \kappa^Q = 2.0$$

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Contact Us

website: www.quant-next.com

email: contact@quant-next.com

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